QCHAPTER 8
CONCLUDING REMARKS

The primary concern of this thesis is to use statistical methods to solve important engineering applications, namely, the comparison of two digital images. Comparing of two images is done by comparing their respective feature vectors. Regarding both vectors as being subject to errors, ideas of regression, in particular the MULFR model is considered. The correlation-based similarity measures R_F^2 and R_p^2 were developed from the ULFR and MULFR models and used as a performance indicator. Maximum likelihood estimators of the parameters of the MULFR model were considered. The estimators of $\hat{\boldsymbol{\alpha}}$ and $\hat{\beta}$ were shown to be unbiased, consistent and asymptotical normal. Since $R_p^2 = \dfrac{\hat{\beta} S_{xy}}{S_{yy}}$, all good properties of $\hat{\beta}$ will be reflected in R_p^2. The properties of R_p^2 is intent investigated in a simulation study. The versatility of $\hat{R}_p^2$ was shown in its successful applications in a data compression problem and a character recognition problem. Highlight of this study is attempted to give a physical interpretation to R_p^2 in a given application. R_p^2, for example, can be used as a performance indicator for compression problem. This chapter ends with discussions of topics for further research followed by a list of publications related to the study.

8.1 Research Conclusion

8.1.1 Findings of Literature Review

A reviewed of more than 80 research papers related to SISM shows that this study is the first to propose MULFR model, and to use it for solving image processing problems. It is a generalization of the ULFR model to *p*-dimensional.

The chronological development of statistical measures from year 1980 to year 2010 was investigated. Table 2.1 provides the list of image processing applications that involved the process of comparing two images. These applications vary from image segmentation to video quality assessment. A significant interest on statistical measure started after year 1990 and it comprised about 25% of the total number of similarity measures for comparing two images. These similarity measures are classified into six categories based on different statistical concepts such as correlations, moments, information theory and non-parametric. It showed the development trend of the statistical similarity measures and its strengths and limitations in various image processing applications.

Three issues for designing a good similarity measure were identified from the literature review. These issues are imperfect full reference, multiple image features and local versus global information. It was showed that most of the ISMs perform well to perfect reference image, single image feature and comparing images either locally or globally. In many studies, each of these issues has been treated separately with one or more similarity measures. It is essential to consider a imperfect reference image since most of the digitized images contained various types of noise or manipulations by nature. It is fragmentary to consider only single feature since an image is represented by various features. Both the local values of an image and the whole image carried different information and it is necessary to measure the image at the local and global levels together. One important contribution of this study is that R_p^2 exhibits good properties and robust measure when subjected to these constraints.

8.1.2 Theoretical Properties of R_p^2

The derivation of a new similarity measure started with the unreplicated linear functional relationship (ULFR) model. The essential different between ULFR model

and the conventional regression models is that ULFR model assumes both regressors and response variable are subject to errors. This assumption makes ULFR model accords with only the need for a non-perfect reference image. It's coefficient of determination, R_F^2 was derived as a measure for comparing two images.

The ULFR model was then extended to its multidimensional version called MULFR model with single slope. This new MULFR model is suitably tailored to some image processing applications, which considered the three constraints together in a particular measure. The coefficient of determination R_p^2 was developed as a generalized version of R_F^2.

The maximum likelihood method was applied for parameters estimation. The maximum likelihood estimators were shown to be unbiased, consistent, efficient and asymptotically normal. The confidence intervals were obtained. Properties of R_p^2 were also discussed, which satisfied the criteria for a good ISM, namely simplicity, boundedness, reflexivity (identity of indiscernible), reaction to enlightening/darkening (translation invariant), reaction to binary images (nonnegative), prediction monotonicity, prediction consistency and prediction accuracy. Although this similarity measure is generally not symmetry, it is not an essential requirement in the proposed applications and it does not obviously affect the experimental results. In order to overcome this problem, we regard the reference image or the distorted image, whichever has larger variance as $\boldsymbol{X}$, and the other image as $\boldsymbol{Y}$. With this arrangement, the full range $0 \le R_p^2 \le 1$ will be granted. Otherwise, Equation (5.35) provides a simple way to convert $\tilde{R}_p^2$ to R_p^2. A symmetry measure can also be easily obtained by taking the average of $\tilde{R}_p^2$ and R_p^2.

8.1.3 Simulation to Verify Sampling Properties of $\hat{\alpha}$, $\hat{\beta}$ and R_p^2

The sampling properties of $\hat{\boldsymbol{\alpha}}$, $\hat{\beta}$ and R_p^2 were studied under two situations. Firstly, the sampling properties when the assumptions or conditions for the MULFR model are not satisfied. Secondly, even if all assumptions are satisfied, the performance of R_p^2 need to be investigated when particular errors or restrictive conditions may be imposed on the manner of handling or manipulating the images. One example used in this study is the Laplacian noise generated by JPEG compression process which follows a Laplace distribution. The simulation study discussed the cases when both errors are normal or only one error is not normal because the reference images are subjected to white Gaussian noise which has a normal distribution.

Two types of image structures were considered in the simulation study to mimic the real data, namely simple and complicated image structures. When both errors are normally distributed, the performance of MULFR model and R_p^2 are not affected by the sample size and image structures. For a fixed sample size, the performance of the model and R_p^2 drop as the ratio between the two error variances is greater than 20.

Under the assumptions of normality and the ratio of error variances equal to one, all estimators are robust and unbiased for both simple and complicated data structures. A good estimation can still be obtained for small sample size of 10 and λ not more than 20. When one of the errors violated the normality assumption, the performance of R_p^2 remains good for small error standard deviations, but the under estimation problem was presented after $\sigma = 10$, especially for complicated data structure. are found to be robust for a small sample size of 10 and the error variance is less to 10. In general, R_p^2 performan better under simple data structure with smaller average confidence intervals. It was also observed that the larger the sample size, the smaller the confidence interval size.

8.1.4 Application in Character Recognition

The first application is the problem of matching online handwritten Chinese characters with a given database. The Chinese character recognition techniques are well known as complicated problem due to the complex structure of characters, large shape variation, large character set and many instances of highly similar characters. In this study, a novel online handwritten Chinese character recognition system was developed. This character recognition system consists of a new database with only one sample per class/character. The feature extraction method utilized the *X*-graph, *Y*-graph and Haar wavelet transformations. The largest similarity value between the handwritten character and the character in database will mean that the handwritten character has been matched or recognized as the database character.

The new character recognition system was found to be robust to the size, slant and position variation of the handwritten character. By using R_p^2, the new character recognition system showed several advantages as compared with other matching methods such as R_F^2, R_S^2, MSSIM, RMSE, CDBB, CMF, MQDF and MD. For instance, the R_p^2 gave among the highest recognition rates and precision for different writers' experience and character complexities. It achieved at least 94% recognition rates even the pre-processing stage is removed from the recognition system.

Efficiency is another important ingredient of a good recognition system. The R_p^2 is clearly superior to other matching methods by significantly reduced at least 40% of the processing time of the recognition system. As a conclusion, the character recognition system with R_p^2 is accurate, precise, efficient and robust to different writing styles. This is not impossible to achieve because R_p^2 allows variations (errors) in both *x*-direction and *y*-direction of the handwritten character.

8.1.5 Application in Image Compression

Image compression is an important engineering area covering a broad range of related applications from communications to entertainment. One key issue in image compression is to evaluate the quality of the compressed image. The similarity measures R_F^2 and R_p^2 served as quality measure, and hence the performance indicator by comparing the compressed image with the full reference image. The use of R_F^2 and R_p^2 requires that the reference image and the compressed image be treated as non-perfect, both being subject to errors. Additionally, the R_p^2 also considered multiple image features from both the local and global information of an image, nevertheless, only luminance and contrast values were found to be useful in this study. According to Wang and Chen (2000), the luminance and contrast are the key factors that affect visual performance and display quality of visual display terminals such as television. Where as, Frazor and Geisler (2006) found that there is a weak correlation between local luminance and contrast in natural images. This supported our selection that both luminance and contrast are essential consideration in image quality study.

Experiments showed that the proposed similarity measures performed as good as other ISMs under the criteria of monotonicity, accuracy and consistency at high compression factors. At low JPEG compression factors, it was shown that R_p^2 is more sensitive and conformed to human's judgement based on Mean Opinion Score (MOS). The confidence interval of R_p^2 is a good indicator of which compression factor will produce reliable quality value. Other properties of R_p^2 were shown to be boundedness, reflexivity, translation invariant and nonnegative. These additional properties provide advantages on the application of R_p^2, for example, the nonnegativity of R_p^2 allowed meaningful interpretation of relationship, as compared to MSSIM which can take negative values.

This study appears to be the first to introduce a consistent and simple interpretation of the image quality by relating the similarity measure to the percentage of distorted area. With this relationship, it is possible to estimate the amount of distortion generated by a compression technique from its similarity value, regardless of the types of image tested. In the JPEG compression application, it will be shown that the proposed similarity measure also work well for Reduced Reference approach when some information in the reference image are lost (subjected to noise).

8.2 Areas of Further Research

There are several areas that could be extended for further study. These areas are as follow:

8.2.1 Study the MULFR Model with Correlated Errors

The MULFR model assumes errors or departures are identical and uncorrelated. Removing the assumption that $\boldsymbol{\delta}_i$ and $\boldsymbol{\varepsilon}_i$ are uncorrelated may be considered in future research. Sprent (1969) loosen these error restrictions and dealt with estimation problems in the more general multivariate and multivariable situations. The solution of this problem invokes the principle of generalized least squares method. Fisk (1966) and Malinvaud (1966) had considered some special patterns of correlation of departures, and Chan & Mak (1984) considered heteroscedastic errors in multivariate linear functional relationship. Further analytical and simulation studies can be conducted for MULFR model which involve correlated errors for both homoscedastic normal model and heteroscedastic normal model. Correlated errors can be introduced into these distributions and the impact of model dimensions, sample size, ratio of error variances and departure from normality can be investigated.

8.2.2 Different Regression Methods

The study of the MULFR model can be extended to include non-linear model or other forms of relationship. Huxley (1924) transformed the non-linear data using logarithm and studied the linear relationship between the transformed log x and log y. While in recent years, Hussin (2001, 2005) and Hussin et al. (2003, 2005) considered the bivariate circular functional relationship model under unreplicated, pseudo-replicates and replicated situations. The errors of circular variables are independently distributed and it follows the von Mises distribution with zero mean and known ratio of the error concentration parameters. Besides, nonparametric MULFR model can also be explored where Ferraty & Vieu (2006) discussed the simple functional nonparametric regression model in bivariate case.

8.2.3 Exploring More Applications of R_p^2 in Image Processing

Two applications of R_p^2 were discussed in this study. They are image quality assessment for JPEG compression and classification (matching method) for online Chinese character recognition system. The following are some possible areas in image processing that R_p^2 can be applied. More potential applications of R_p^2 such as image registration and medical treatment using X-ray were also discussed in Rijal, et al. (2011).

(i) Detection of Image Watermarking

Digital image watermark is an identification code or distinguishing piece of information that is permanently embedded into digital image (Zheng et al., 2007). It is intended to hide proprietary information in digital media for copyright protection, distribution tracing, authentication and conditional access control. An example of watermark embedding process for still image is given in Figure 8.1. The original (host) image $\boldsymbol{X}$ is passed through a perceptual analysis to identify alterable pixels so that the

resulting watermarked image is indistinguishable from the original (Perez-Gonzalez & Hernandez, 1999). This process yielded a perceptual mask. The message to be hidden is then shaped by the perceptual mask which is controlled by a secret key to protect the message. It is difficult to recover the hidden information without knowing this secret key. The hidden message can be text or picture. Finally, the watermark is spread and embedded to the original image yielded the watermarked image ***Y***. Both original image and watermarked image can be saved in JPEG or other formats.

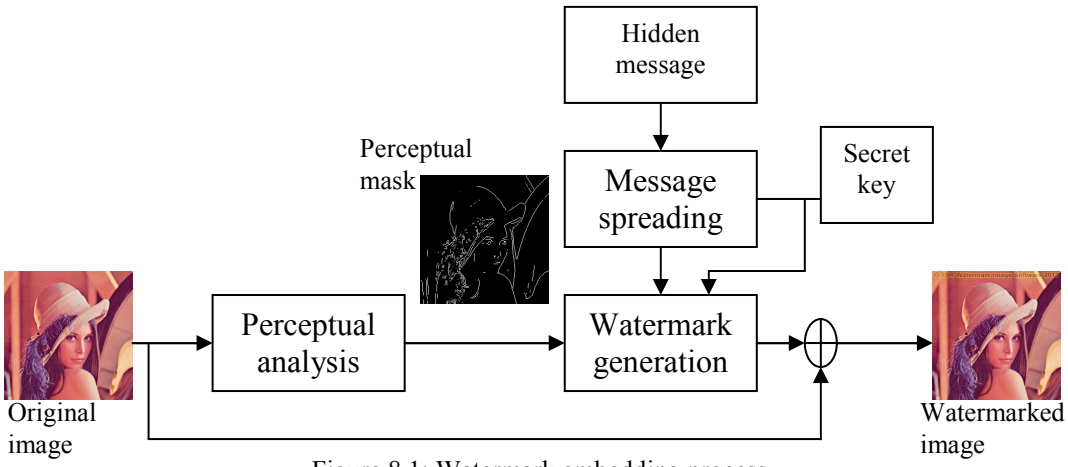

Figure 8.1: Watermark embedding process

The watermarking process can be expressed mathematically. A watermarked image ***Y*** is an embedding function $e(\bullet)$ who takes the watermark ***W***, the original image ***X***, and the security key ***K*** as an input parameters (Zheng et al., 2007).

$$\boldsymbol{Y} = e(\boldsymbol{X}, \boldsymbol{W}, \boldsymbol{K}) \tag{8.1}$$

An example of embedding function is $\boldsymbol{X} + \boldsymbol{W}(\boldsymbol{M})$ where the additive watermark ***W*** carries a perceptual-mask ***M*** (Voloshynovskiy et al., 2001) which is the hidden message.

One important issue of a watermark is imperceptibility. A digital watermark is imperceptible if the original image and the marked image are perceptually indistinguishable (Perez-Gonzalez & Hernandez, 1999). The watermark is called perceptible if its presence in the marked data is identifiable, but non-intrusive.

The proposed R_p^2 can be used to evaluate the imperceptibility of watermarking by comparing the watermarked image $\boldsymbol{Y}$ and the original image $\boldsymbol{X}$. In this case, the watermark $\boldsymbol{W}$ can be considered as error $\boldsymbol{\varepsilon}$ in the image $\boldsymbol{Y}$ and the original image $\boldsymbol{X}$ is assumed to be subjected to error $\boldsymbol{\delta}$ as stated in Equation 4.1 and Equation 4.2. R_p^2 should indicate a large similarity value when watermarking is imperceptible and has a small similarity value for perceptible watermarking.

As a preliminary experiment, the TSR Watermark Image Software 1.9.9.4 was used to embed the text "@ TSR Watermark Image Software 2011" at the center of the image obtained from the image database USC_SIPI. The two images used are 'Couple' and 'Saiboat' images. The free watermark software allows controlling the imperceptibility of the watermark with the transparency from 0% to 100%. Transparency at 100% means the watermark is imperceptible, and the perceptibility increase as the transparency level decrease. Figure 8.2 shows the examples of host images 'Couple' and 'Saiboat', and their corresponding watermarked images at 100%, 60% and 0% transparencies. The luminance and contrast values obtained from the watermarked image and host image were used as the input to calculate the R_p^2. Other ISMs use the intensity values as their input.

Figure 8.2: Watermarked Couple image and Saiboat image. Host image, 100%, 60% and 0% of transparency (from left to right).

Figure 8.3 compares the similarity values of ISMs against the percentage of transparency of watermark. At high transparency levels above 70%, all selected ISMs have large similarity value indicates that the watermark is almost imperceptible. R_p^2 is more sensitive than other selected ISMs at moderate and low transparency levels below 70%. MSSIM, R_S^2 and R_F^2 remain large similarity values when the watermark is perceptible, while R_p^2 value decreases gradually when the transparency of watermark decreases.

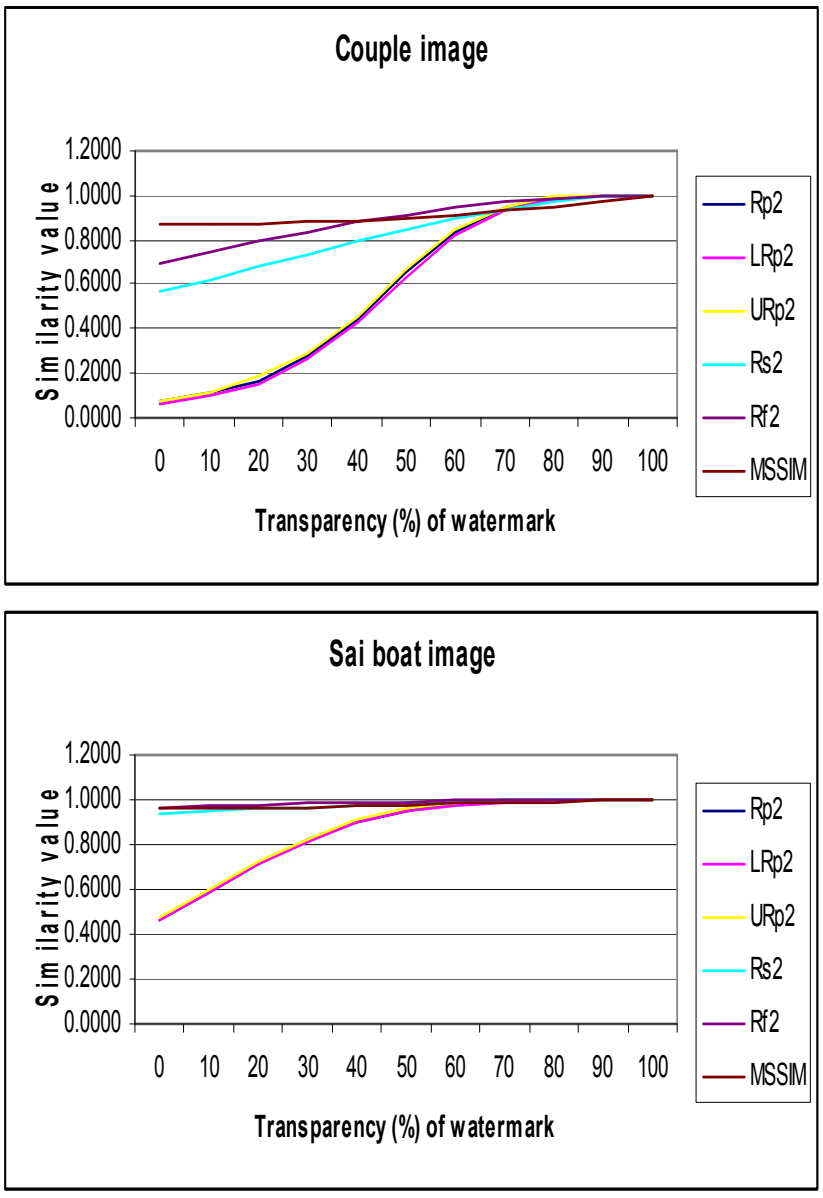

Figure 8.3: Comparing the similarity values of ISMs against the percentage of transparency of watermark. LRp2 and URp2 are lower and upper confidence limits of Rp2.

The achievement of R_p^2 in watermark detection problem is not as good as the performance in character recognition and JPEG compression problems. One possible reason is the luminance and contrast used as feature vector are not appropriate for watermarking. It is believed that the performance of R_p^2 in watermarking perceptibility evaluation can be further improved if a more appropriate feature vector is defined and used.

(ii) Video compression assessment

The application of R_p^2 in JPEG compression image quality assessment can be extended to video compression quality assessment. A video can be viewed as a sequence of images or frames with size $M \times N \times T$ where $M \times N$ is the size of each frame and T is the total number of frames considered (see Figure 8.4). The matrix $\boldsymbol{x} = [x_{ki}]$ where x_{ki}, $k = 1, 2, \ldots, T$; $i = 1, 2, \ldots, MN$ can be constructed from the intensity or a group of intensity values of a compressed video and the reference video yields $\boldsymbol{y} = [y_{ki}]$ where y_{ki}, $k = 1, 2, \ldots, T$; $i = 1, 2, \ldots, MN$. The R_p^2 can be used to measure the overall quality of the compressed video, instead of calculating the average quality from each frame.

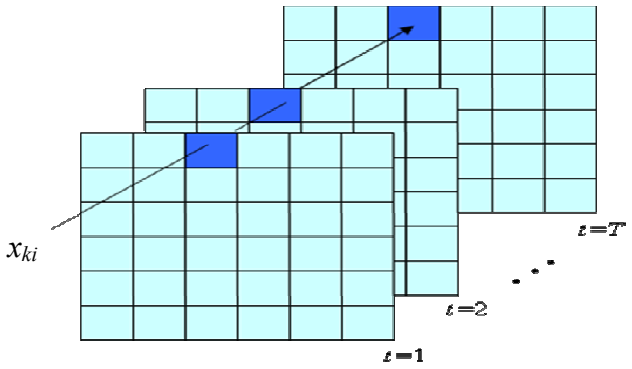

Figure 8.4: A sequence of video frames.

(iii) Face recognition

Face recognition system plays a crucial role in security and identification purposes. It is used to identify a person by comparing the input image with the database images. A person can be said to be "identified" if both input and database facial images are similar to each other. One of the applications in face recognition is in forensic and criminal case investigation called photo fit system. It enables eyewitness to construct an image of the criminal suspect by composite different set of face features, such as eye brow, eye, nose, lip and ear (Shi et al., 2006). In order to preserve salient face features, Wong et al. (2007) used landmark based face model to construct feature extraction. Anatomical landmark which contains points that has biological significance is applied (Shi et al., 2006). These points play a crucial role to differentiate the dissimilarity between two subjects. Some points are necessary to be emphasized in order to preserve enough face information. Figure 8.5 shows the anatomical points' location and its distance towards centre point as proposed by Wong et al. (2007). Nevertheless, there is always a research interest to find a better way of defining face features.

The centre point, denoted by (a_0, b_0), is calculated by taking the centre of inner points of the eyes, $P_1(a_1, b_1)$ and $P_2(a_2, b_2)$:

$$\text{Centre point} = (a_0, b_0) = \left(\frac{a_1 + a_2}{2}, \frac{b_1 + b_2}{2} \right) \tag{8.2}$$

Euclidean distance and angle of a point (a_i, b_i) are used to determine the differences between human features and is calculated with respect to the centre point as follows:

$$\text{Distance: } D_i^x = \sqrt{(a_i - a_0)^2 + (b_i - b_0)^2} \tag{8.3}$$

$$\text{Angle: } \theta_i^x = \tan^{-1}\left(\frac{b_i - b_0}{a_i - a_0} \right) \tag{8.4}$$

for (a_i, b_i), $i = 1, 2, \ldots, n$. Each input image can be reduced into a two-dimensional matrix as follows:

$$\boldsymbol{x}_i = \left(\text{distance}_i, \text{angle}_i\right) = \left(D_i^x, \theta_i^x\right) \tag{8.5}$$

and the database image yields $\boldsymbol{y}_i = \left(D_i^y, \theta_i^y\right)$. Then, R_p^2 between the input image and the database images can be calculated. The R_p^2 measures the relationship between two variables and determines how close are the information of the input image compared to the database image.

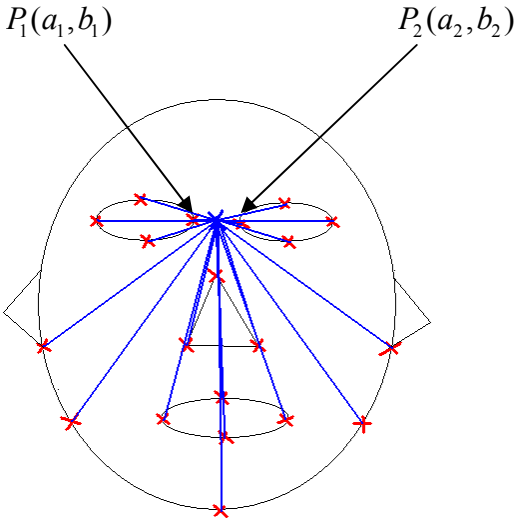

Figure 8.5: Location of anatomical points and distance with centre point (reference point)

8.3 Published Articles

During the course of this study the following related works were published:

(i) Referred Journal

1. Chang Yun Fah, Abdul Ghapor Hussin and Omar Mohd Rijal, An Investigation of Causation: The Unreplicated Linear Functional Relationship Model, Asian Network for Scientific Information: Journal of Applied Sciences, Vol. 7, No. 1, pp. 20 – 26, 2007. (SCOPUS-Cited Publication).
2. Chang Yun Fah, Omar Mohd Rijal and Syed Abdul Rahman Abu Bakar, Functional Quality and Performance Metric for Some Image Processing

Applications, NAUN International Journal of Mathematical Models and Methods in Applied Sciences, Vol. 2, No. 4, 2008, pp. 543 – 552. (SCOPUS-Cited Publication).

3. Chang Yun Fah, Omar Mohd Rijal and Syed Abdul Rahman Abu Bakar, Multidimensional unreplicated linear functional relationship model with single slope and its coefficient of determination, WSEAS Transactions on Mathematics, Vol. 9, No. 5, May 2010, pp. 295 – 313. (SCOPUS-Cited Publication).
4. Chang Yun Fah, Lee Jia Chii, Omar Mohd Rijal and Syed Abdul Rahman Syed Abu Bakar, An efficient handwritten Chinese character recognition system using 2-dimensional functional relationship model, International Journal of Applied Mathematics and Computer Science. Vol. 20, No. 4, 2010, pp. 727 – 738. (ISI-Cited Publication).

(ii) Conference Proceedings

1. Chang Yun Fah, Omar Mohd Rijal and Syed Abdul Rahman Abu Bakar, Functional Image Quality and Performance Evaluation for JPEG Compression and De-Noising Filters, Canary Islands: Proceeding of the 13th WSEAS International Conference on Applied Mathematics, 2008, pp. 113 – 118.
2. Lee Jia Chii, Fong Tzu Jen and Chang Yun Fah, Feature Extraction for Handwritten Chinese Character Recognition Using X-Y Graphs Decomposition and Haar Wavelet, Kuala Lumpuer: Proceedings of the IEEE International Conference on Signal and Image Processing Applications (ICSIPA2009), 18-19 November 2009.

3. Chang Yun Fah, Lee Jia Chii, Tong Wei Loo and Gan Fei Sin, A New Classifier for Handwritten Chinese Character Recognitin Using 2-Dimensional Functional Relationship Model, Shanghai: Proceedings of the IEEE International Conference on Intelligent Computing and Intelligent Systems (ICIS2009), Vol. 4, 20-22 November 2009, pp. 1 – 4.
4. Omar Mohd Rijal, Norliza Mohd Noor and Chang Yun Fah, Application of Correlation as a Measure of Performance, Amman, Jordan: The 4th IEEE International Symposium on Inovation in Information and Communication Technology (ISIICT2011), 29 Nov – 1 Dec 2011. Accepted.